

MAGNETOHYDRODYNAMIC BOUNDARY LAYER FLOW OVER A STRETCHING SHEET WITH CHEMICAL REACTION

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ABSTRACT:

An analysis has been made to investigate the effect of chemical reaction on the magneto hydrodynamic flow and heat transfer over a stretching sheet. The surface velocity of the stretching sheet and the transverse magnetic field are assumed to vary as a linear function of the distance from the origin. The effect of internal heat generation is taken into account. A generalized similarity transformation is used to reduced the governing partial differential equations to a system of non-linear coupled ordinary differential equations, and is solved numerically by using a finite difference scheme. The numerical results concerned with the velocity, temperature and concentration distributions for various values of the dimensionless parameters of interest. Some important findings reported in this project reveal that the effect of magnetic field and chemical reaction have significant role in controlling the rate of heat transfer in the boundary layer region.

INTRODUCTION:

The boundary layer flow over a stretching sheet has been the recent topic of researchers and engineers, because of its many engineering and industrial applications. The investigation of the boundary-layer flows and heat transfer of an incompressible fluid over a stretching surface has also many important applications such as the extrusion of plastic sheets from a die, the boundary layer along a liquid film condensation process, gas turbines, MHD power generators, the cooling process of metallic plate in a cooling bath, in the glass and polymer industries, flight magneto hydro dynamics as well as in the field of planetary magnetosphere, aeronautics and chemical engineering. The rate of heat transfer at the stretching sheet plays a vital role to obtain better quality of the final product. The effects of mass transfer with chemical reaction on the boundary layer flow has enormous applications in chemical engineering processes such as to enhanced oil recovery, packed-baked catalytic reactors, solidification of binary alloy as well as catalytic surface reactions in hydrodynamic flows Sakiadis (1961) initiated the study of boundary layer flow on a continuous moving surface. Later on Crane (1970) extended this problem to a stretching sheet whose surface velocity varies linearly with the distance x from the fixed point. Consequently Gupta and Gupta (1977) examined the heat and mass transfer over a linear stretching sheet subjected to suction /blowing. The influence of uniform magnetic field on the flow of an electrically conducting fluid passed a stretching sheet have been investigated by Pavlov (1974), Anderson (1992), Watanabe and Pop (1993, 1995), Shit (2009), Prasad et al.(2010) and Shit and Haldar (2012a). Afify (2004) investigated the chemical reaction on free convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching sheet in the presence of a uniform transverse magnetic field. Misra and Shit (2008) extensively studied on electrically conducting fluid flow and heat transfer in a parallel plate channel with stretching walls in the presence of a magnetic field applied externally. Two-dimensional unsteady MHD flow of a viscous fluid between two moving parallel plates has been investigated by Sweet et al. (2011). Jafar et al. (2012) carried out the steady magneto-hydrodynamic (MHD) laminar boundary layer flow of a viscous and incompressible electrically conducting fluid near the stagnation point on a horizontal

stretching or shrinking surface with variable surface temperature and a constant magnetic field applied normal to the surface of the sheet Vajravelu and Cannon (2006) numerically examined the effects of nonlinearly stretching sheet on the flow characteristics of viscous incompressible fluid. Prasad et al. (2013) have investigated the effects of internal heat generation/absorption, thermal radiation, magnetic field and temperature dependent thermal conductivity on the flow and heat transfer characteristics of a Non-Newtonian fluid over a stretching sheet. To characterize the non-Newtonian fluid behaviour they have used upper convected Maxwell (UCM) fluid model. The numerical results for the physical variables have been obtained by using finite difference scheme along with the Keller-Box method. Mustafa et al. (2011) carried out the magneto hydro dynamic boundary layer flow and heat transfer of an electrically conducting micro polar fluid over a non linear stretching surface with variable wall heat flux in the presence of heat generation/absorption. Shit and Haldar (2011b) numerically investigated the effects of thermal radiation and variable fluid properties on the MHD fluid flow over a non-linear inclined stretching sheet with variable viscosity in the presence of heat generation/absorption. The viscous dissipation and buoyancy effects are taken into account in a situation when there is a temperature dependent viscosity .The present problem pertains to a situation in which the chemical reaction takes place. Since the governing differential equations are highly non-linear, Newton’s linearization method is used followed by the finite difference scheme to obtain numerical solutions. The numerical results of the flow characteristics are presented graphically.

MATHEMATICAL FORMULATION AND FLOW ANALYSIS:

Let us consider the steady two-dimensional MHD flow and heat transfer along with the chemical reaction phenomenon of an incompressible, viscous and electrically conducting fluid past over a stretching sheet. A uniform strong magnetic field of strength B_0 is imposed along the perpendicular to the sheet. The x-axis is assumed to be the direction of the flow and y-axis is normal to it. The temperature and the species concentration are maintained at a prescribed constant value T_w and C_w at the sheet and T_∞ and C_∞ are the fixed values far away from the sheet. The continuous stretching surface is assumed to have a power-law velocity $u = U_w = bx$, where $b > 0$ is a constant, x denotes the distance from the slit, n is the power index and U_w represents the surface velocity of the sheet. We consider that electrically conducting fluid is influenced by the applied magnetic field of strength B_0 normal to the stretching sheet. We assume that the induced magnetic field produced by the motion of an electrically conducting fluid is negligible, since there is no electric field because of the negligible polarization of charges. The effect of induced magnetic field can also be neglected because of the low electrical conductivity of the fluid which in turn produces low magnetic Reynolds number. The boundary layer flow over a non-linear stretching sheet is governed by the following system of equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \dots\dots\dots(1)$$

$$\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = \mu \frac{\partial^2 u}{\partial y^2} + \rho_\infty g_0 \beta_t (T - T_\infty) + \rho_\infty g_0 \beta_c (c - c_\infty) - \sigma B_0^2 u \dots\dots\dots(2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g_0 \beta_t (T - T_\infty) + g_0 \beta_c (c - c_\infty) - \frac{\sigma B_0^2}{\rho_\infty} u \dots\dots\dots(3)$$

$$\rho_\infty c_\beta \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_0^2 u^2 \dots\dots\dots(4)$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2} - k_0 (c - c_\infty) \dots\dots\dots(5)$$

where u and v are the velocity components along the x and y directions respectively, μ is the coefficient of viscosity, g_0 the acceleration due to gravity, β_t , the coefficient of thermal expansion, β_c the coefficient of expansion with concentration α the angle of inclination of the stretching sheet with the horizontal line. T and C are the temperature and concentration variable respectively D the thermal molecular diffusivity, k_0 is the reaction rate constant, C_p is the specific heat at constant pressure, k is the thermal conductivity, T_∞ and ρ_∞ are the free stream temperature and density.

The boundary conditions for the present problem can be written as

$$u = u_w = bx, v = 0, T = T_w, c = c_w \text{ at } y = 0 \quad \dots\dots\dots(6)$$

$$u \rightarrow 0, T \rightarrow T_\infty, c \rightarrow c_\infty \text{ when } y \rightarrow \infty \quad \dots\dots\dots(7)$$

Now we introduce the following similarity transformations as

$$\psi = \sqrt{bv}xf(\eta)$$

$$\eta = \sqrt{\frac{b}{v}}y$$

$$\theta(\eta) = \frac{(T-T_\infty)}{(T_w-T_\infty)} \quad \dots\dots\dots(8)$$

$$\phi(\eta) = \frac{(c - c_\infty)}{(c_w - c_\infty)}$$

$$u = \frac{\partial \psi}{\partial y}$$

$$v = -\frac{\partial \psi}{\partial x}$$

where ψ and f represent the dimensional and non dimensional stream function respectively

Where $\nu = \frac{\mu}{\rho_\infty}$ the kinematic viscosity, η is a similarity space variable in non

dimensional form,

θ and ϕ are the dimensionless temperature and concentration.

Automatically by u and v defined in Eq (8).Substituting Eq(8) in Eqs(2) to (7) yields

$$f'''(\eta) + f(\eta)f''(\eta) - f'^2(\eta) + \lambda_t \theta(\eta) + \lambda_c \phi(\eta) - Mf'(\eta) = 0 \quad \dots\dots\dots(9)$$

$$\theta''(\eta) + P_r f \theta' = -(P_r E_c f'^2 + M P_r E_c f'^2) \quad \dots\dots\dots(10)$$

$$\phi'(\eta) + Sc f(\eta)\phi' - Sc v \phi(\eta) = 0 \quad \dots\dots\dots(11)$$

Where the parameters are

$$\lambda_t = \frac{g_0 \beta_t (T_w - T_\infty)}{b^2 x}, \quad \lambda_c = \frac{g_0 \beta_c (c_w - c_\infty)}{b^2 x}$$

$$M = \frac{\sigma B_0^2}{b \rho_\infty}, \quad P_r = \frac{\rho_\infty c_p v}{k}$$

$$E_c = \frac{u^2}{c_p (T_w - T_\infty)}, \quad Sc = \frac{\mu}{\rho_\infty D} = \frac{\nu}{D} \text{ since } \nu = \frac{\mu}{\rho_\infty} = \frac{k_0}{b}$$

Boundary conditions are taken as

$$f'(0) = 1, f(0) = 0, \theta(0) = 1, \phi(0) = 1$$

$$f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty$$

RESULTS AND DISCUSSION:

We observe from Fig1 that the axial velocity in the boundary layer region decreases gradually with the increase of the Prandtl number Pr. We observe from Fig2 that the axial velocity in the boundary layer region decreases gradually with the increase of the magnetic field strength. This may attribute to the Lorentz force that arises due to the application of an external magnetic field in an electrically conducting fluid. This force has a tendency to slow down the motion of the fluid and thereby decreases momentum boundary layer thickness. We observe from Fig3 that the concentration of species in the boundary layer decreases for increasing values of the parameter γ . Thus the parameter γ is responsible for the reducing of concentration boundary layer thickness

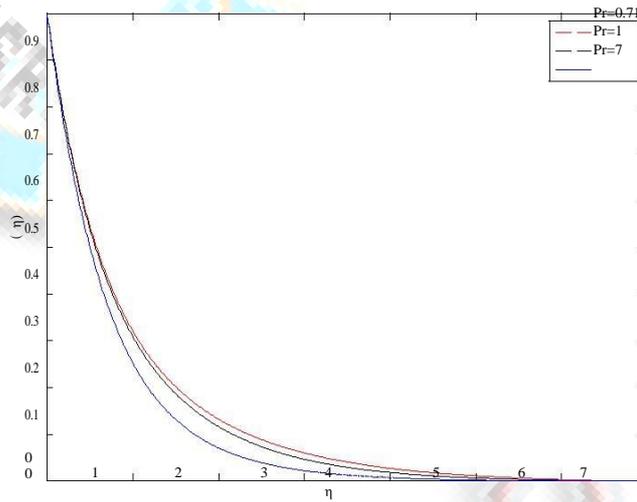


FIG. 1: The density parameter ρ is shown against r

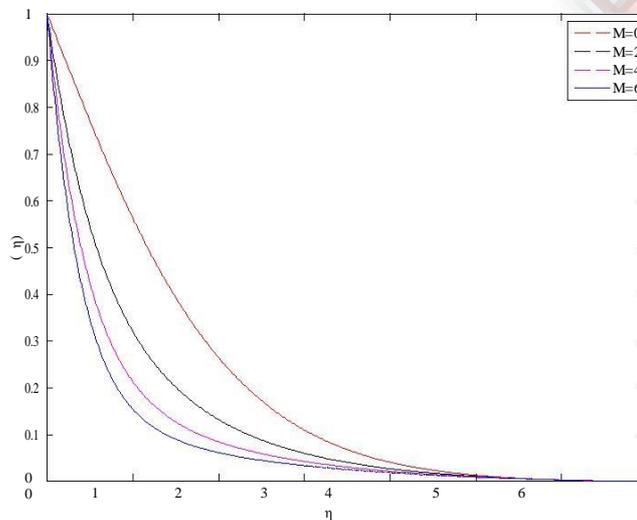


FIG. 2: Variation of $f'(\eta)$ with η for different values of M.

From Fig4 it is seen that the concentration of species in the boundary layer decreases with the increase of the magnetic field strength. Fig5 depicts the temperature distribution in the boundary layer for the variation in Grashoff number Gr_t . It is observed that the temperature has decreasing effect with the increasing values of Gr_t . Fig6 illustrates the distribution of dimensionless temperature $\theta(\eta)$ along the height from the stretching sheet for different values of M. It is seen that temperature increases with the increase of the magnetic field. Fig 7 illustrates the distribution of dimensionless temperature $\theta(\eta)$ along the height from the stretching sheet for different values of

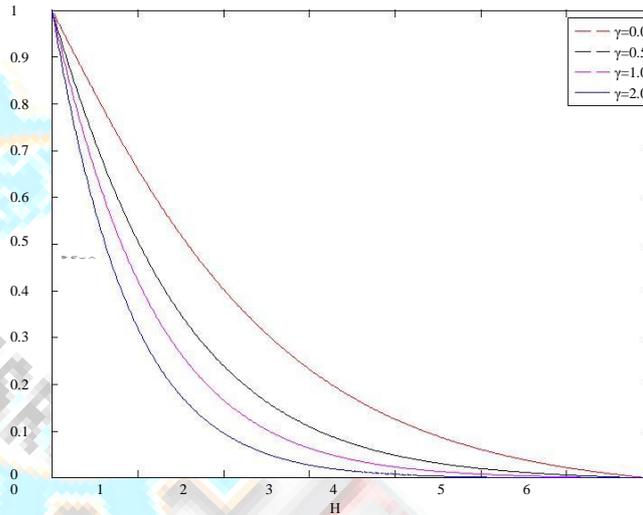


FIG. 3: Concentration profiles for different values of chemical reaction parameter γ .

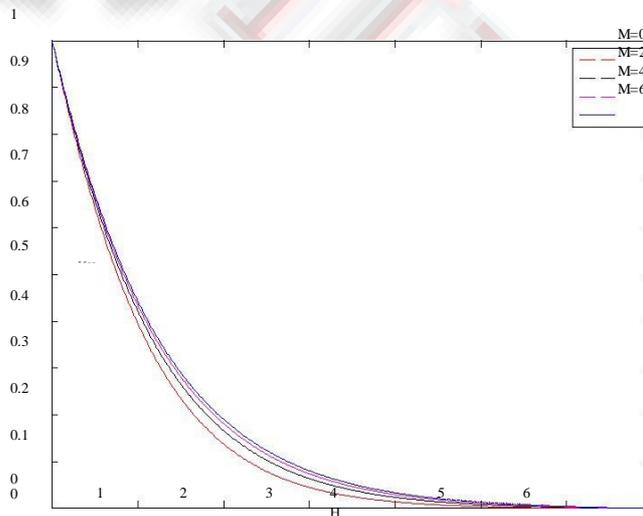


FIG. 4: Concentration profiles with η for different values of M .

Pr. The effect of Prandtl Pr on the heat transfer is shown in fig7. It reveals that the temperature is also significantly decreases with the increase of the Prandtl number Pr .

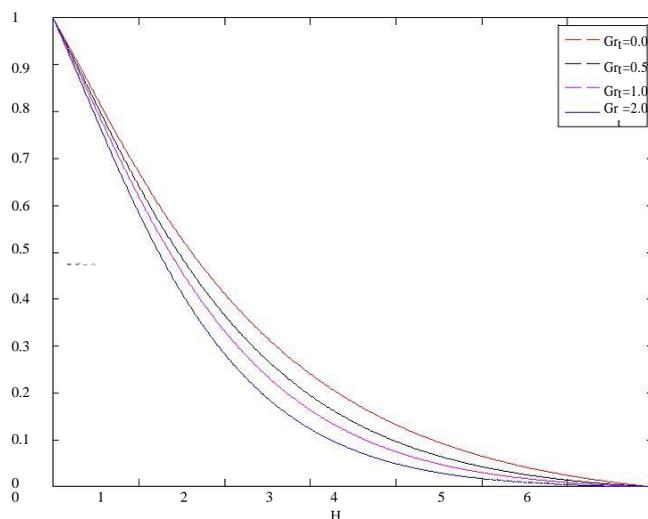


FIG. 5: Distribution of dimensionless temperature $\theta(\eta)$ for different values of Gr_t .

REFERENCES:

1. Anjalidevi, S.P. and M. Kayalvizhi (2013). Nonlinear hydromagnetic flow with radiation and heat source over a stretching surface with prescribed heat and mass flux embedded in a porous medium. *Journal of applied fluid Mechanics*, 6(2), 157-165.
2. Gupta, P.S and A.S Gupta (1977). Heat and Mass Transfer on a stretching sheet with suction or blowing. *The Canadian Journal of Chemical Engineering*, 55, 744-746.
3. Misra, J.C and G.C. Shit (2009b). Flow of a bio magnetic viscoelastic fluid in a channel with stretching walls. *ASME Journal of Applied Mechanics*, 76, 061006.
4. Mostafa, A.A. Mahmoud and S.E. Waheed (2011). MHD flow and heat transfer of a micro polar fluid over a nonlinear stretching surface with variable surface heat flux and heat generation. *The Canadian Journal of Chemical Engineering*, 89, 1408-1415.
5. Pavlov, K.B. (1974). Magneto hydrodynamic flow of an incompressible viscous fluid caused by deformation of plane surface. *Magnitnaya Gidrodinamika*, 4, 146-147.
6. Sakiadas, B.C (1961). Boundary layer behaviour on continuous solid surfaces. *American Institute of Chemical Engineering Journal (AIChE)*, 7, 26-28.
7. Shit, G.C and R. Halder (2012b). Combined effects of thermal radiation and hall current on MHD free convective flow and mass transfer over a stretching sheet with variable viscosity. *Journal of Applied Fluid Mechanics*, 5(2), 113-121.
8. Shit, G.C. and R. Halder (2011b). Effects of thermal radiation on MHD viscous fluid flow and heat transfer over nonlinear shrinking porous sheet. *Applied Mathematics and Mechanics*, 32, 677-688.
9. Shit, G.C. and R. Halder (2012a). Thermal radiation effects on MHD viscoelastic fluid flow over a stretching sheet with variable viscosity. *International Journal of Applied Mathematics and Mechanics*, 8, 14-36.
10. Vajravelu, K and C.R Cannon (2006). Fluid flow over a nonlinearly stretching sheet. *Applied Mathematics*.